Tutorial 7 - Fundamental Interactions

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Jul 2023

1 Mass spectrum

Consider a complex scalar field with potential

$$V = -m^2(\phi^2 + (\phi^*)^2) + \lambda(\phi^*\phi)^2, \tag{1}$$

with m^2 and λ positive.

a) Rewrite the potential in terms of two real fields. Find the eigenvalues of the Hessian of V evaluated at the origin. How can one interpret this result?

b) Find the minimum of the potential. Shift the minimum to the origin and rewrite the potential in terms of the shifted fields, then find the new eigenvalues of the Hessian at the origin.

2 SSB of global G-symmetric scalar theory

Consider ϕ a triplet of real scalar fields and the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi^T\partial_\mu\phi - V(\phi)\,, \quad \text{with} \quad V(\phi) = \frac{\lambda}{4!}\left(\phi^T\phi - v^2\right)^2\,. \tag{2}$$

a) What is the explicit global symmetry group G in the Lagrangian above? Break this symmetry by choosing a particular vacuum and shifting it to the origin, then rewrite the new Lagrangian. How many massive and massless scalars do we have now?

b) Show that the new Lagrangian still has some remaining symmetry. Which one is it? Check the consistency of your results with Goldstone's theorem.

3 SSB of SU(2) YM theory with scalar in the fundamental representation

Consider a SU(2) Yang–Mills theory coupled through the covariant derivative to a scalar doublet ϕ transforming in the fundamental representation of SU(2). Let $V(\phi) = -m^2|\phi|^2 + \lambda(|\phi|^2)^2/4$.

a) Where is the minimum of the potential above reached? Choose a particular vacuum ϕ_0 and use it to shift the minimum of V to the origin.

b) After the shift, find the explicit A^2 (with no derivatives) term of the Lagrangian. How many of the gauge bosons acquired mass? What are their masses?

4 SSB of SU(2) YM theory with scalar in the adjoint representation

Repeat the previous exercise, but now with ϕ being a scalar triplet transforming in the adjoint representation of SU(2).

5 Georgi-Glashow GUT model

The gauge group of the Standard Model of particle physics is $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where C refers to "color", L comes from "left", and Y refers to "hypercharge". Here, $\mathrm{SU}(3)_C$ is the gauge group of QCD, and $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is the gauge group of the electroweak theory, which unifies the weak and the electromagnetic interactions. A theory that tries to unify strong and electroweak interactions as a single interaction at high energies is called a Grand Unified Theory (GUT for short). It embeds the Standard Model gauge group into one larger simple group. As $\mathrm{SU}(3)_C\times$ $\mathrm{SU}(2)_L\times$ $\mathrm{U}(1)_Y$ has rank 4, any GUT has to be based on a semi-simple Lie group of rank at least 4. There is only one such group with rank 4 that is compatible with the strong and electroweak interactions, the group SU(5). One can also study groups with higher ranks, for example SO(10), which has rank 5, but for this exercise we will stick to SU(5).

The SU(5) Georgi-Glashow model has a scalar field in the adjoint (24-dimensional) representation, $\phi = \phi^a\, t_a$. Its effective (energy-dependent) potential is chosen to develop a nontrivial minimum below a certain energy, where the field acquires a vacuum expectation value proportional to the weak hypercharge generator, that is, $\phi_{\text{vac}} = \frac{1}{6} v_{24}\,\mathrm{diag}(-2, -2, -2, +3, +3)$, whose stabilizer subgroup is $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) \subset \mathrm{SU}(5)$.

a) Explain how $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is mapped (up to a $\mathbb{Z}_6$ factor) into $\mathrm{S}(\mathrm{U}(3) \times \mathrm{U}(2)) \subset \mathrm{SU}(5)$.

b) With the information given above, compute the number of massive and massless gauge bosons in this model, after the SSB.

c) Does this GUT predict as many massive gauge bosons as the Standard Model has? The new gauge bosons are called X and Y bosons, and they combine to produce interactions that are not present in the Standard Model, as quark-quark and quark-lepton annihilation. Those violate baryonic number conservation, which doesn't agree with experiments we have today. What is a possible explanation as to why the Georgi-Glashow SU(5) GUT cannot be ruled out for this fact?